The Simple Analytics of the Private Provision of Public Goods

A Diagrammatic Exposition

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$$U_i(x_i, G)$$

Pure Public Good: G

1. Non-rival consumption—i.e., consumption by one agent does not reduce consumption by another.
2. Non-excludable consumption—i.e., everyone consumes the same amount (technnical characteristic).

Make all prices equal to 1 (by choosing units appropriately). Optimality (Samuelson condition) requires:

$$\sum_i \frac{\frac{\partial U_i(x_i^\star, G^\star)}{\partial G}}{\frac{\partial U_i(x_i^\star, G^\star)}{\partial x}} = \sum_i MRS_i(x_i^\star, G^\star) = 1 = MRT$$

The Bergstrom-Blume-Varian (BBV) Model

Simplest possible model: one-shot non-cooperative game

- Agent i's endowment: w_i
- Can be consumed as private good: x_i
- Can be contributed towards the public good: g_i
- Total public good: $G = \sum_i g_i = g_i + \sum_{j \neq i} g_j = g_i + G_{\sim i}$

Agent i solves:

$$\begin{aligned} \max_{x_i, g_i} \quad & U_i(x_i, G) \\ \text{s.t.} \quad & x_i + g_i = w_i \\ & G = g_i + G_{\sim i} \\ & x_i, g_i \geq 0 \end{aligned} \tag{1}$$

Nash Equilibrium in a Private Contributions Game

(One-shot non-cooperative game

- The optimization problem (1) can be restated as:

$$\max_{g_i} \left\{ U_i(w_i - g_i, g_i + G_{\sim i}) \ : \ 0 \leq g_i \leq w_i \right\} \tag{2}$$

- Each agent i chooses her contribution, g_i, taking others' as given $G_{\sim i}$
- Private comsumption is a residual $x_i = w_i - g_i$

Nash Equilibrium

A vector of contributions $\{g_i^*\}$ is a Nash equilibrium of the private contributions game if g_i^* solves (2) for all i with $G_{\sim i} = \sum_{j \neq i} g_j^*$

Nash Equilibrium in a Private Contributions Game

Properties of NE

- Existence and Uniqueness
- Suboptimality—since $MRS_i = 1 \Rightarrow \sum_i MRS_i = n$ while optimality requires $\sum_i MRS_i = 1$; underprovision

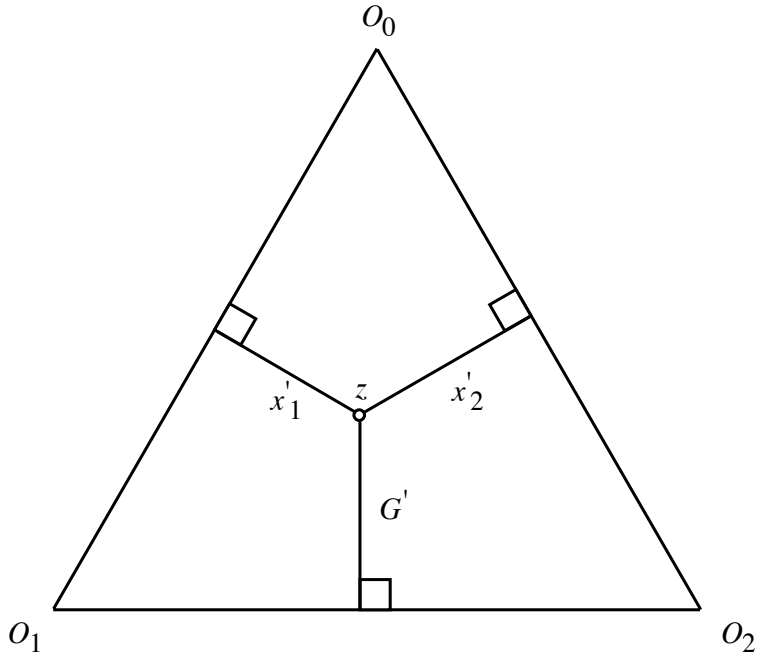
Other results

- Neutrality with respect to small redistributions (Warr 83)
- Regressive redistribution may be welfare enhancing (Itaya, Meza & Myles 97)

The Kolm Triangle: allocations

Economy: 2 agents, 2 goods (1 private, 1 public good)

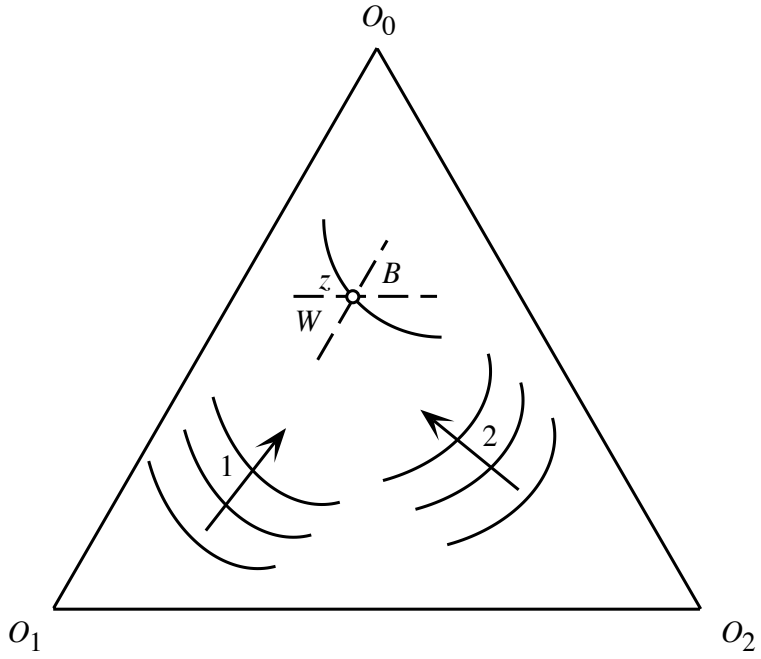
Aggregate resource constraint: $x_1 + x_2 + G = W =$ constant.



A feasible allocation (x'_1, x'_2, G') in the Kolm triangle.

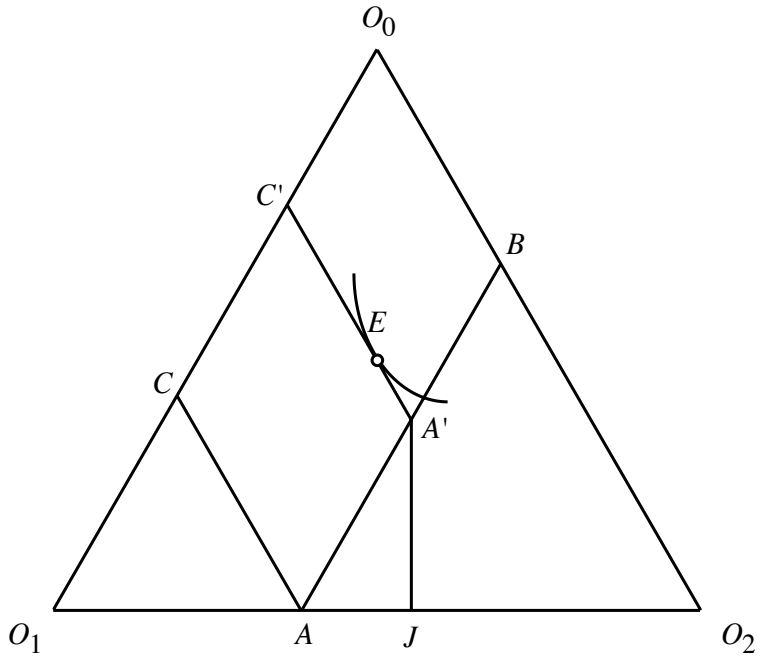
The Kolm Triangle: preference maps

Economy: 2 agents, 2 goods (1 private, 1 public good)



$B(z)$ are allocations strictly Better than z.
$W(z)$ are the allocations strictly Worse to z.

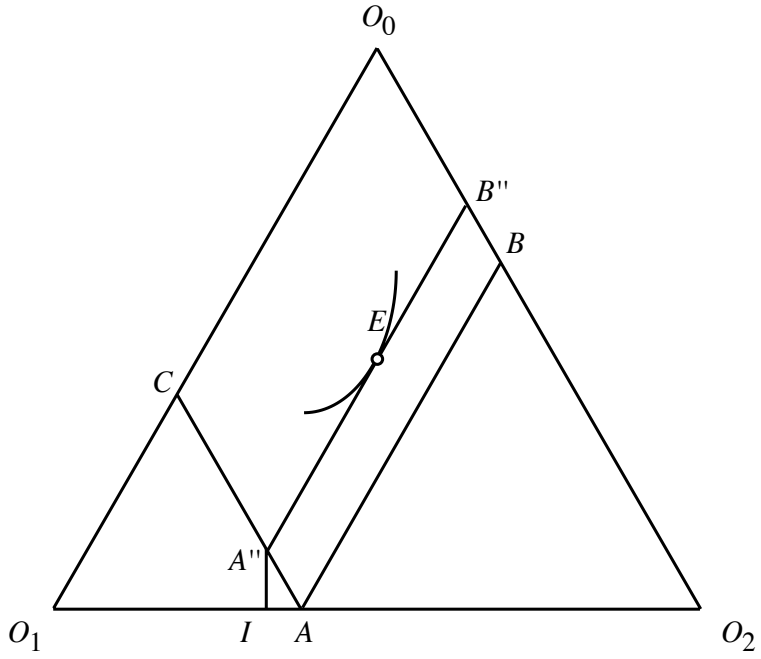
The Kolm Triangle: Agent 1's viewpoint



Taking agent 2's contribution $g_2 = JA'$ as given,
agent 1 maximizes her utility at E.

The Kolm Triangle: Agent 2's viewpoint

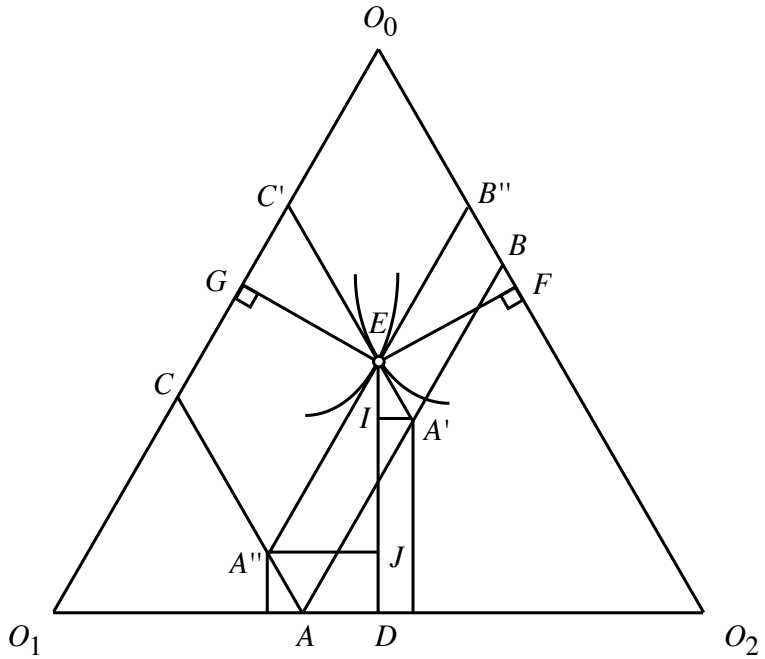
Economy: 2 agents, 2 goods (1 private, 1 public good)



Taking agent 1's contribution $g_1 = IA''$ as given, agent 2 maximizes his utility at E.

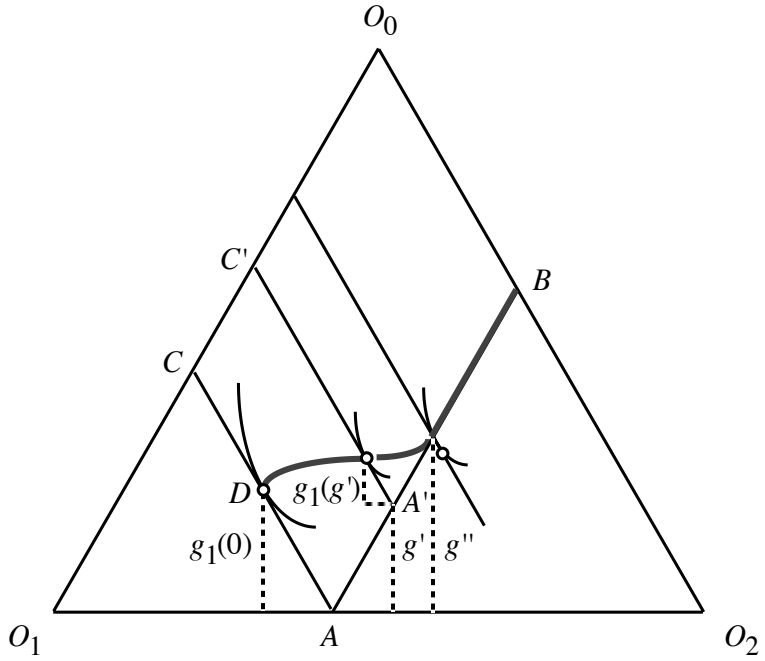
The Kolm Triangle: Suboptimality of Equilibrium

Each agent is maximizing her utility given the other agent's actions.



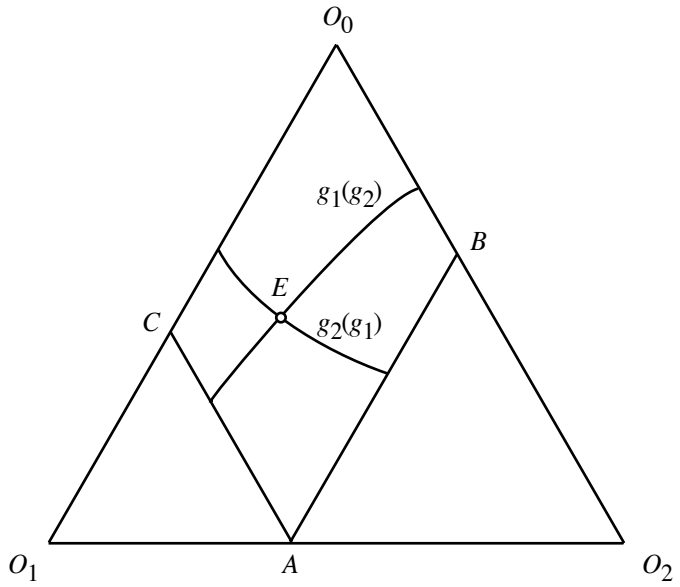
The Kolm Triangle: Agent 1's reaction function

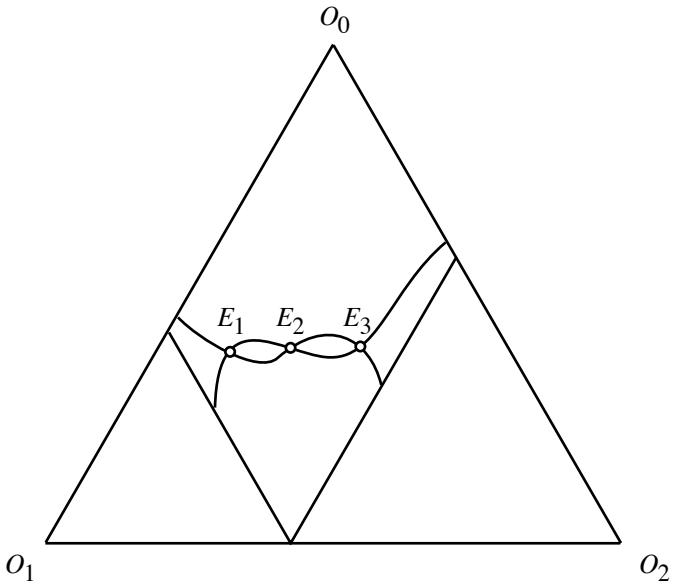
$$g_1(g) \equiv \text{argmax}_{g_1} \left\{ U_1(w_1 - g_1, g + g_2) \; : \; 0 \leq g_1 \leq w_1 \right\}$$



Note corner solutions, $g_1(g_2) = 0$ once $g_2 > g''$.

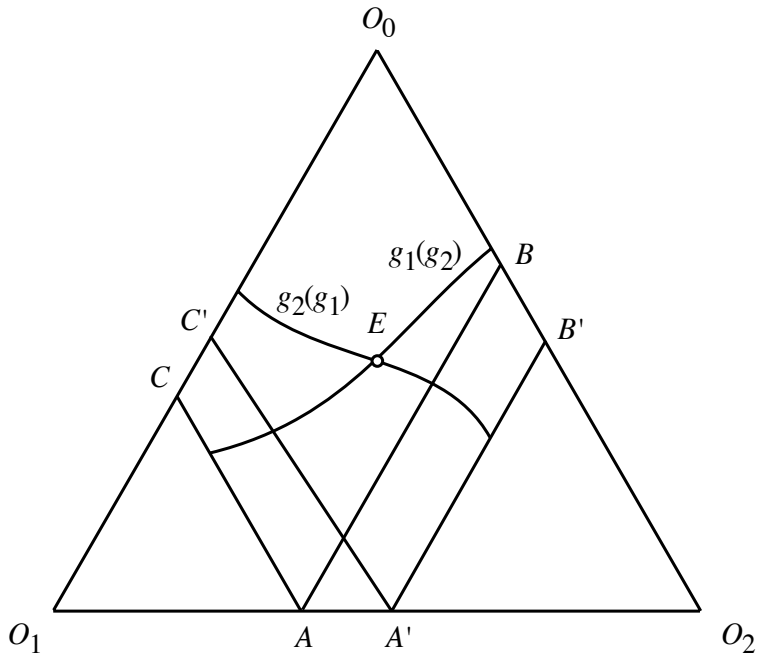
Uniqueness of the Nash Equilibrium



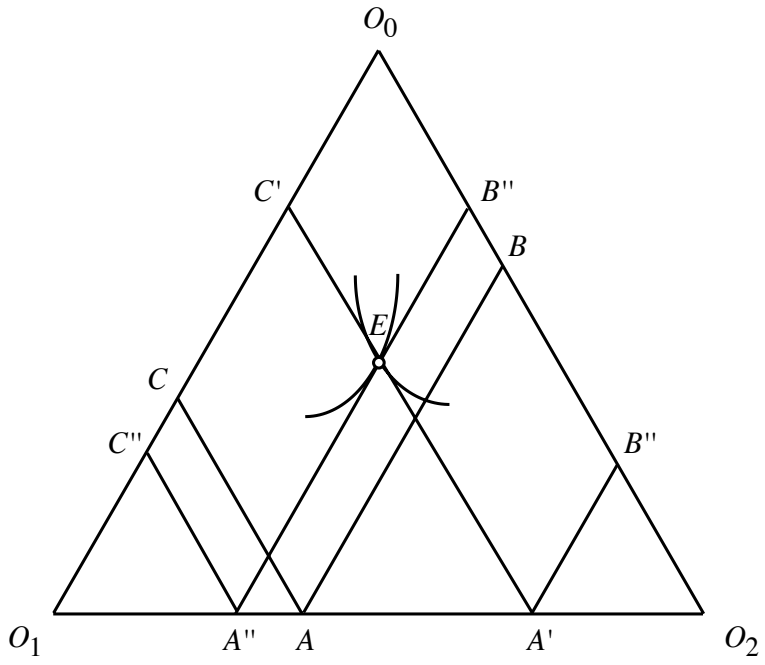


Non-negative slope follows from normality of x and G.

Neutral Redistribution (Warr)



Limits on Neutral Redistributions



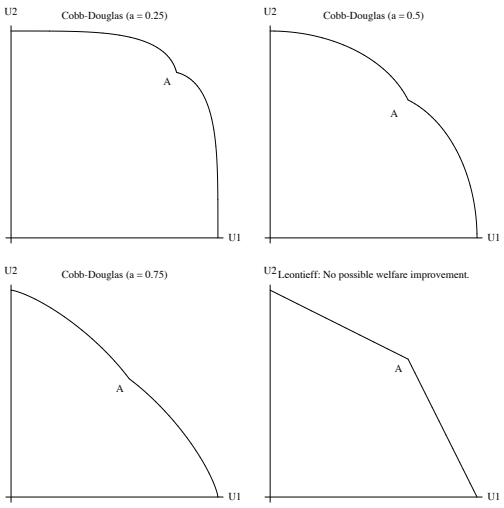
Regressive Redistributions are Welfare Enhancing

Social Welfare Increased by Increasing Inequality

- Social welfare can be raised by creating sufficient income inequality so that only the richer can afford to contribute to the public good.
- Intuition: Private consumption of poorer agent is substituted by a mix of private consumption and public-good provision by the richer agent.
- Scope for welfare-enhancing regressive redistributions depends on:
  - the individual preferences (utility)
  - the social preferences (SWF)

Utility Possibility Frontier—Redistributions

Reference: symmetric pre-redistribution Nash equilibrium A



$U_i = x_i^a G^{1-a}$ $U_i = \min\{x_i, aG\}$

Social Welfare Function:

- Utilitarian $W = U_1 + U_2$ (SICs are straight lines)
- Rawlsian $W = \min\{U_1, U_2\}$ (SICs are Leontieff)

The Advantage of Moving First

The Stackelberg equilibrium will be determined by the leader, 1, choosing her most preferred point the follower's, 2, reaction function:

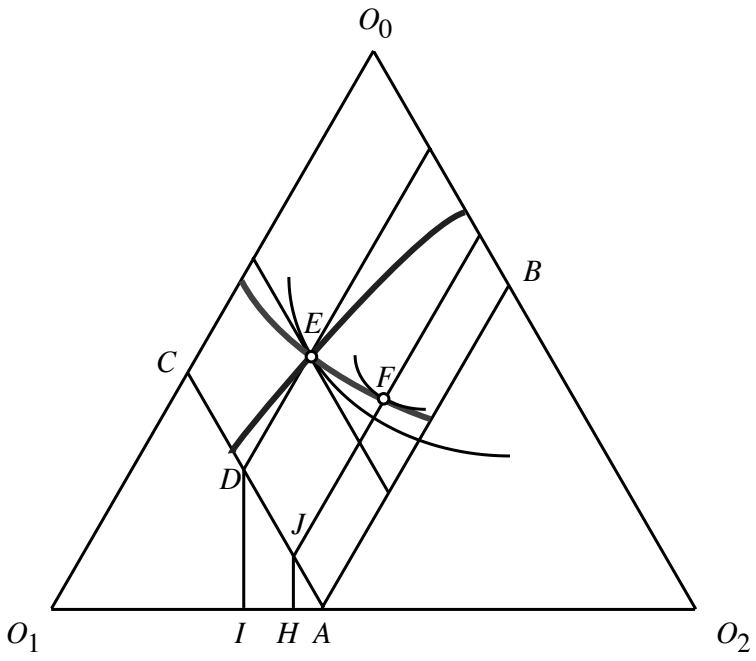
$$\max_{g_1} \quad \{ U_1(w_1 - g_1, g_1 + g_2(g_1)) \ : \ 0 \leq g_1 \leq w_1 \}$$

Results

1. $g_1^s \leq g_1^n \quad \rightarrow$ leader contributes less
2. $G^s \leq G^n \quad \rightarrow$ total amount is smaller
3. $g_2^s \geq g_2^n \quad \rightarrow$ follower contributes more (homework)

Preliminaries The Kolm Triangle BBV Results Changing the Game Optimal Outcomes Other References

The Advantage of Moving First



Subsidizing Further Contributions from NE

MRS at NE

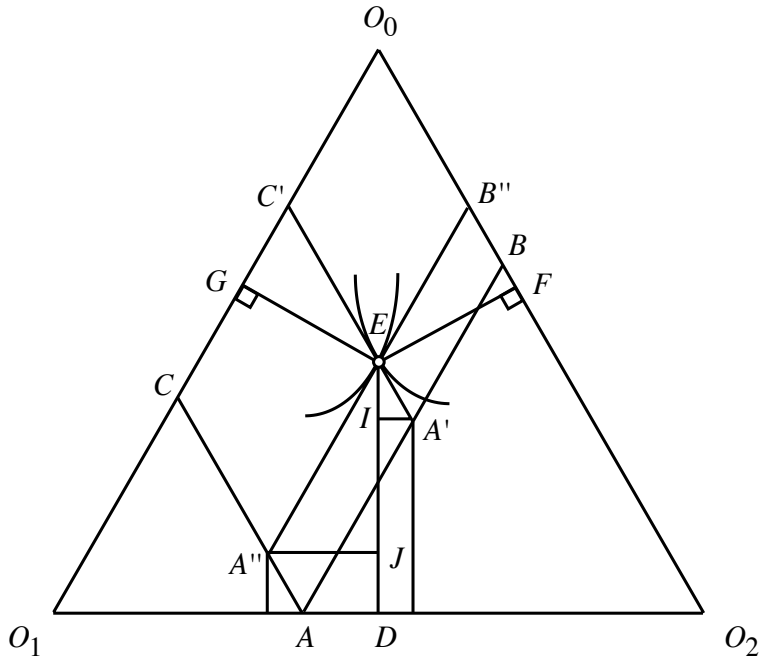
At the Nash Equilibrium, each agent is happy with the level of G, given what it costs:

$$MRS_1(x_1^*, G^*) = \frac{\frac{\partial U_1(x_1^*, G^*)}{\partial G}}{\frac{\partial U_1(x_1^*, G^*)}{\partial x}} = 1 = \frac{\frac{\partial U_2(x_2^*, G^*)}{\partial G}}{\frac{\partial U_2(x_2^*, G^*)}{\partial x}} = MRS_2(x_2^*, G^*) \quad (3)$$

Agent 1 subsidises half the cost to agent 2 of an extra unit of G

Both agents are better off—they virtually each gets to purchase an extra unit for half the price that they were paying before.

Subsidizing Further Contributions from NE



Question: What about agent 1 subsidising all contributions made by 2?

Please—Do not subsidise me!

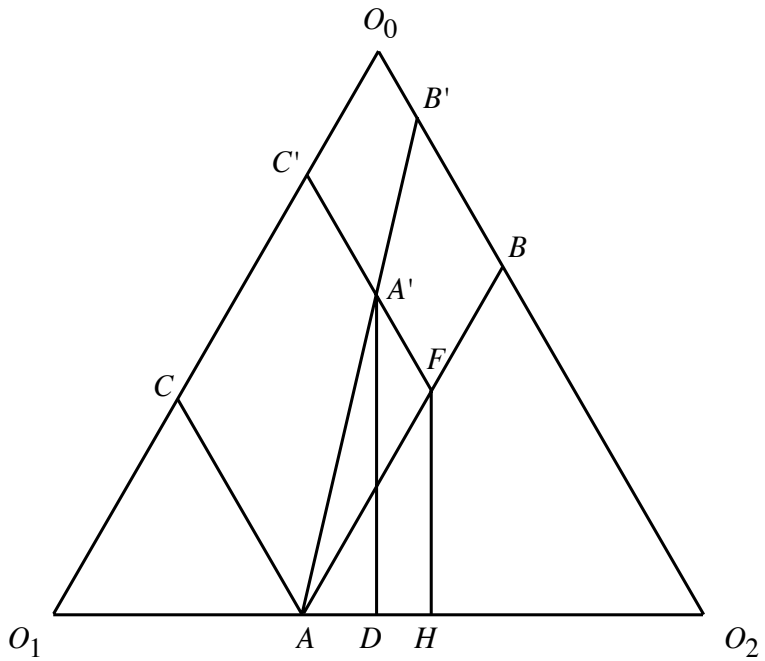
When Subsidies are Allowed—Puzzling Results

- Rich people might be made worse off when their contributions are subsidised at a higher rate than poor people—eg, when the contributions are tax-deductible in a system of progressive income taxation (Roberts 1987).
- Choosing the right subsidy rate:
  - It is advantageous to subsidise the other agent, even from the very first unit contributed.
  - The person receiving the subsidy is made worse off.

  (Boadway, Pestieau & Wildasin 1989).

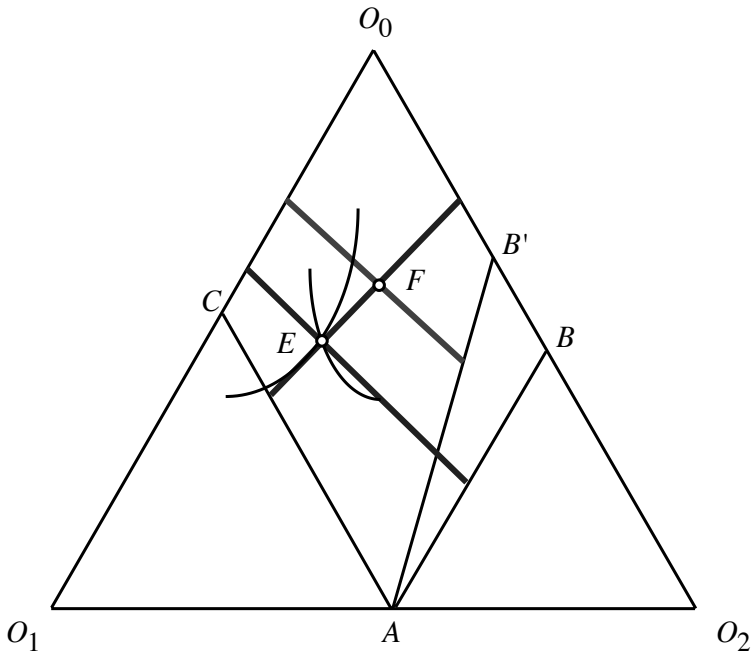
Subsidizing Contributions—Geometry of a Subsidy

Agent One subsidises Agent Two—Isn't Agent Two's Feasible Set Enlarged?



Recipient of Subisidy: Worse Off

Agent Providing Subsidy: Better Off



$$G^F > G^E \qquad x_1^F > x_1^E \qquad x_2^F < x_2^E$$

Two-Stage Games

One-Subsidy Game

Agent one will subsidize agent two at a pre-determined rate $s \in (0,1)$:

$$\max_{g_2} \left\{ U_2(w_2 - (1-s)g_2, g_1 + g_2) \ : \ 0 \leq (1-s)g_2 \leq w_2 \right\} \qquad (4)$$

$$\max_{g_1} \{ U_1(w_1 - sg_2 - g_1, g_1 + g_2) \ : \ 0 \leq g_1 \leq w_1 - sg_2 \} \qquad (5)$$

At the equilibrium:

$$MRS_2(x_2^*, G^*) = (1-s) \quad \text{and} \quad MRS_1(x_1^*, G^*) = 1 \qquad (6)$$

The two agents are now facing different (personalized) prices.

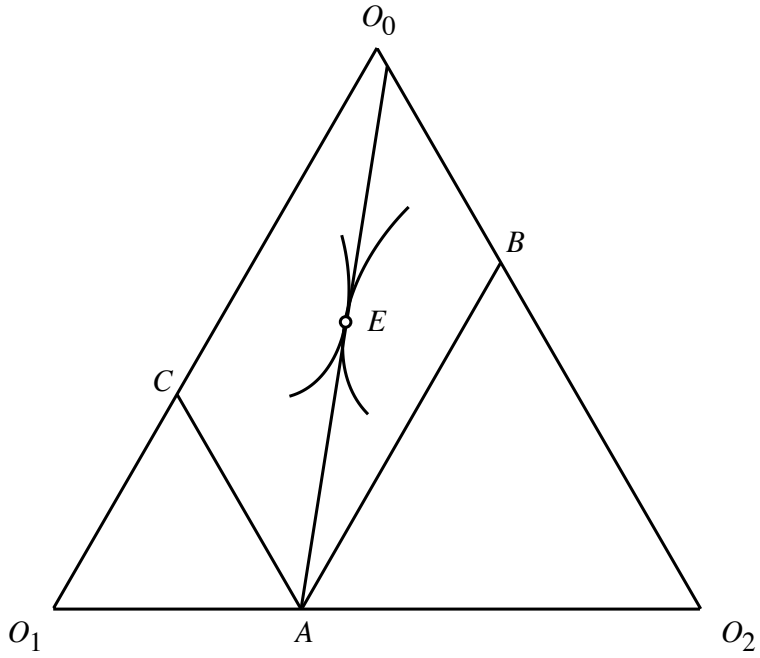
Lindhal Equilibrium—Agents face personalized prices

- Given $\mathbf{p} = (p_1, p_2) > 0$, with $p_1 + p_2 = 1$ (feasibility)
- Each agent i computes her demand for G:

$$G_i(\mathbf{p}) = \arg\max_G \left\{ U_i(w_i - p_i G, G) \ : \ w_i - p_i G \geq 0 \right\} \tag{7}$$

- Equilibrium: Given $\mathbf{p}^*$, we have $G_1(\mathbf{p}^*) = G_2(\mathbf{p}^*)$
- Optimality: We have $MRS_1 + MRS_2 = p_1 + p_2 = 1$
- Note duality to Walrasian equilibrium—where same prices, different consumption bundles.

Lindahl prices partition the Kolm triangle into two disjoint budget sets with a common boundary:



Lindahl equilibria are Pareto optimal—no room for improvement.

Two-Stage Subsidy Game—subsidies for all by all

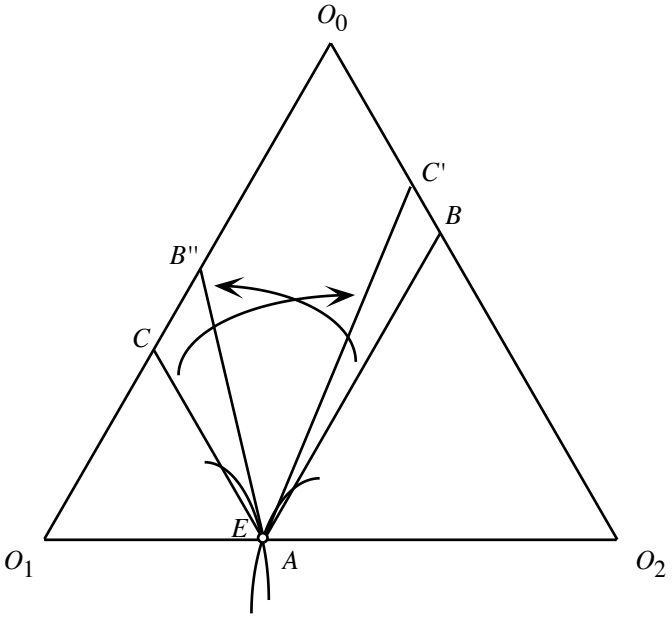
Stage 1: Agent i chooses his $s_j \in [0,1]$ rate for all other j.

Stage 2: Given the total aggregate subsidy that each agent receives from all others, $r_i = \sum_{j \neq i} s_i$, she chooses her contribution to the public good.

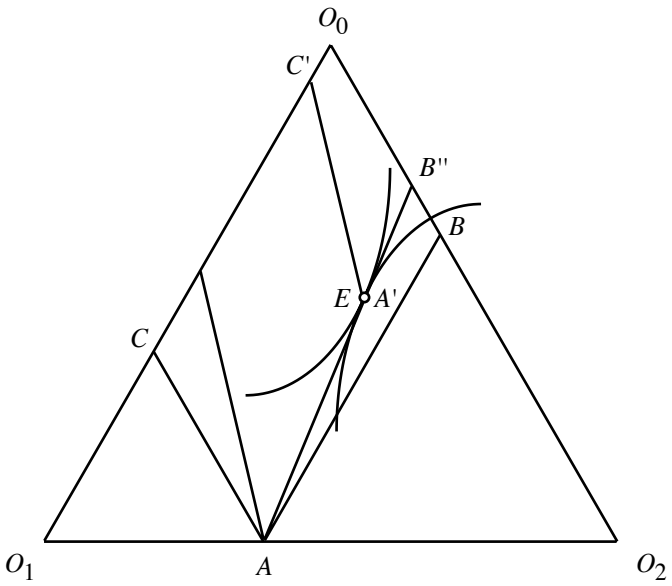
All SPNE of Two-Stage Game correspond to Lindhal Equilibria

1. Inefficient Second-Stage NE cannot support SPNE of Two-Stage Game.
2. The SPNE with $s_1 + s_2 = 1$ are obviously Lindhal Equilibria.
3. Can also have Pareto Efficient SPNE with $s_1 + s_2 \neq 1$.
4. Lindhal equilibria may be supported by different subsidy vectors.

Pareto Efficient Equilibria with $s_1 + s_2 \neq 1$

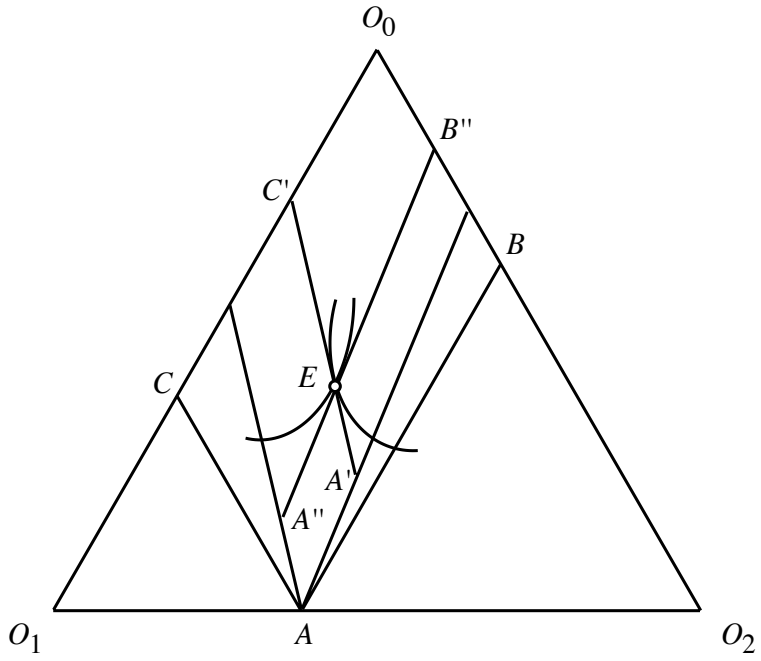


$s_1 + s_2 > 1$



$s_1 + s_2 < 1$
Agent 1 contributes 0

Inefficient equilibrium of 2 stage cannot be SPNE



Each agent would prefer to increase s_i slightly ...

What if G can be provided at different cost?

- Countries differ in their GHG abatement technologies.
- Efficiency requires marginal costs equated across countries—abate where it is cheapest.

$$\max_{g_i} \{ U_i(1 - g_i, w_1 g_1 + w_2 g_2) \ : \ x_i + g_i = 1 \} \tag{8}$$

Rotten Spouse Thm (Konrad and Lommerud, 1995; Ley 1998)

- Spouses can trade leisure for work inside the home (producing household public goods) and outside the home (earning an income).
- Spouse with comparative advantage for out-of-home activities will transfer income to the other—and both are better off because of this.

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